S.V. Akkelin[2], R. Lednicky[1,3] and Yu.M. Sinyukov[1,2]

(28.05.01)

The methods allowing to extract the coherent component of pion emission conditioned by the formation of a quasi-classical pion source in heavy ion collisions are suggested. They exploit a nontrivial modification of the quantum statistical and final state interaction effects on the correlation functions of like and unlike pions in the presence of the coherent radiation. The extraction of the coherent pion spectrum from $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ correlation functions and single–pion spectra is discussed in detail for large expanding systems produced in ultra-relativistic heavy ion collisions.

[1] *SUBATECH, (UMR, Universite, Ecole des Mines, IN2P3/CNRS), 4, rue Alfred Castler, F-44070 Nantes Cedex 03, France.*

[2] *Bogolyubov Institute for Theoretical Physics, Kiev 03143, Metrologicheskaya 14b, Ukraine.*

[3] *Institute of Physics ASCR, Na Slovance 2, 18221 Prague 8, Czech Republic.*

I. INTRODUCTION

The hadronic observables, such as single- or multi-particle hadron spectra, play an important role in the studies of ultra-relativistic heavy ion collisions. However, these observables contain rather indirect information on the initial stage of the collision process since the strong interactions in hadronic matter result in substantial stochastization and thermalization of a system during its evolution. Nevertheless, the final hadronic state could carry some residual signals of the earlier stages of the particle production process. A partial coherence of the produced pions is supposed to be one of the important examples.

The first systematic study of coherent processes in high energy hadron-nucleus $h + A$ collisions was based on Glauber theory [1]. In this theory, the hadron - nucleus collision is considered as a process of subsequent scatterings of the projectile on separate nucleons of the nucleus; the projectile energies are supposed much higher than the inverse nucleus radius ($E_h \gg 1/R$), allowing to consider a linear projectile trajectory inside the nucleus (eikonal approximation). If the scattering process occurred with almost no recoil of the nucleus nucleons, i.e. with no *witnesses* of the individual scatterings, then the hadron-nucleus collision should be described by a coherent superposition of the elementary hadron-nucleon scattering amplitudes. Such a type of the collision is called coherent scattering. In such a scattering, the nucleus does not change its state, and therefore manifests itself just as a particle with some form-factor, usually represented by a Gaussian $\exp(-\mathbf{q}^2R^2/4)$, following from the oscillator approximation of the nucleus wave-function. Clearly, the coherent processes are essential only for small momenta transferred from the projectile hadron to the nucleus: $|\mathbf{q}| < 1/R$. Only then one can neglect the recoil energy and consider the nucleus as a whole during the scattering process. There is a kinematic limitation for the minimal longitudinal momentum transfer $|q_z|_{\min} \approx (M^2 - m_h^2)/(2\,|\mathbf{p}_h|)$, required to produce a particle or a group of particles of the invariant mass M. The vanishing of $|q_z|_{\min}$ with the increasing energy explains why the coherent processes can take place only at high enough energies. It is worth to note, that the integral coherent cross-section does not die out with the increasing energy (see., e.g., [2]).[1]

Typically, however, the transferred momenta are sufficient for substantial *recoil* effects and the excitation of the nucleus or its breakup. The *coherence length* $\sim 1/|\mathbf{q}|$ is small so the nucleus does not participate in the collision as a whole and one can consider the hadron-nucleus collision as an incoherent superposition of elementary hadron-nucleon scatterings, corresponding to random phases of the amplitudes of the latter. The resulting cross-section is then given by the sum of the moduli squared of the amplitudes (probabilities) at each of the possible scattering points (unlike to coherent scattering, when the individual amplitudes are summed up first). As a result, one can expect that particles are produced in chaotic (incoherent) states.

Let us come back to the production of particles (e.g., pions) in the processes of non-elastic coherent scattering at small transferred momenta. Since the nucleus does not excite in these processes and manifests itself as a quasi-classical object, one can describe particle production using the quantum field model of interaction with a classical source [3]. It is well known that the interaction with a classical source results in the production of bosons in coherent states [4], -

[1]We are grateful to V. L. Lyuboshitz for drawing our attention to this important point and for an interesting discussion.

the states which minimize the uncertainty relation and, so, are the closest to classical ones.[2] This is the main physical link between the processes of coherent scattering and particle production in coherent states.

In heavy ion collisions at high energies, the average multiplicities are quite high, e.g., several thousands of pions will be produced at maximal RHIC energies. The inclusive particle spectra thus represent natural characteristics of these processes. A convenient way to account for the coherent properties of these processes consists in a model description of particle emission, rather than in detail evaluation of the contributing amplitudes. The Gyulassy-Kauffmann-Wilson (GKW) model [8] is an example of such an approach. The model assumes that all pions are radiated by classical currents (sources) which are produced in some space–time region during the collision process. Besides the coherent radiation from the individual sources, the model yields also the chaotic component of pion spectra due to the averaging over the space–time coordinates of the source centers. In fact, the chaotic component dominates in case of a large emission region, while, in the opposite limit of very small space–time extent of this region, almost all pions are produced in the coherent state. This seems to be rather general result: if the distances between the centers of pion sources are smaller than the typical wave length of the quanta (the source size), the substantial overlap of the wave packets leads to the strong correlations (indistinguishability) between the phases in pion wave functions and, thus, to the coherence [9,10].

Recently, the coherence of multipion radiation in high energy heavy ion collisions was studied within GKW model in Ref. [11]. In the model, due to the longitudinal Lorentz contraction of the colliding nuclei, almost all pions produced with small transverse momenta $p_t < 1/R$ in central nucleus-nucleus collisions are emitted coherently, and their momentum spectra are determined by the system's space - time extent [11]. Clearly, the coherence of pions can be destroyed by pion rescatterings. Nevertheless, the hadronization time may happen to be large enough to allow a considerable part of the coherent pions escape from the interaction zone without rescatterings [11]. However, as noted in [11], one can expect a strong suppression of the GKW mechanism of coherent pion production if quark-gluon plasma is created: the hadronization then occurs in thermal quark-gluon system and hadrons are produced in the chaotic state only. Note that clear signals of the thermalization and collective flows, observed at CERN SPS and RHIC energies (see, e.g., [12,13] and references therein), point to strong rescattering effects and may reflect also the importance of the quark-gluon degrees of freedom.

The new physical phenomena, which are expected in RHIC and LHC experiments with heavy ions, are associated with the creation of quasi-macroscopic very dense and hot systems. In such systems, the deconfinement phase transition and the restoration of the chiral symmetry are likely to happen, possibly leading to creation of the new states of matter: quark-gluon plasma and disoriented chiral condensate (DCC). In the latter case, another possibility for the coherent pion radiation (above the thermal background) appears. If the DCC were created at the chiral phase transition, a quasi–classical pion field $\vec{\pi}_{cl}$ forms the ground state of the system. The subsequent system decay is accompanied by a relaxation of the ground state to normal vacuum. Such a process can be described by the quantum field model of interaction with classical source (see, e.g. [14]), and results in the coherent pion radiation. One of the general conditions of the ground state rearrangement and formation of the quasi–classical field is a large enough system volume [15]. Therefore, such a field could be generated in heavy ion collisions at sufficiently high energies (RHIC, LHC ?) provided the spontaneous chiral symmetry breaking via DCC formation takes place. The overpopulation of the (quasi) pion medium, making it close to the Bose-Einstein condensation point, can lead to the strengthening of the coherent component conditioned by the classical field decay [16]. Since the quasi-classical pion field appears at the end of the hadronization stage, the coherent radiation could partially survive and be observed.

The most direct way to explore the coherent emission in inclusive measurements is to measure the correlation function $C(p,q)$ of two identical neutral bosons in the region of very small $|q|$; $p = (p_1 + p_2)/2$, $q = p_1 - p_2$. In case of only chaotic contribution, the intercept of the quantum statistical (QS) Bose-Einstein part of the correlation function $C_{QS}(p,0) = 2$ [17] while, in the presence of the coherent radiation, $C_{QS}(p,0) < 2$. Generally, the coherence means strong phase correlations of different radiation components. In Ref. [9], a simple quantum–mechanical model of the phase correlations for one-particle wave packets with different radiation centers has been considered. For a phase–correlated source (corresponding to indistinguishability of the different emitting centers), the emission amplitude $A(p)$ averaged over the event ensemble is not equal to zero, $\langle A(p)\rangle \neq 0$, and the QS correlation function intercept $C_{QS}(p,0) < 2$. In the second quantization representation (more adequate for processes of multi–boson production), the analogous results take place for inclusive averages of the quantum field operators: $\langle a(p)\rangle \neq 0$, $C_{QS}(p,0) < 2$, provided the radiation has a non-zero *coherent state* component. The latter represents a superposition of the states with all possible boson numbers, and with fixed phase relations.

[2] The *coherent states* have been introduced and studied in detail by Glauber [5]. The concept of coherent states was then applied to pion production in high energy processes in Refs. [6–8].

However, charged pions cannot form the usual coherent state, since it obviously violates the super-selection rule. To overcome this difficulty, the generalized concept of charge-constrained coherent states should be used [7,8]. Our consideration is based on the density matrix formalism. In this formalism, the isospin symmetry of the system requires an averaging, in the density matrix, over all orientations of the quasi-classical pion source in the isospin space. As a consequence, the averaged pion field vanishes: $\langle a(p)\rangle = 0$ whereas, for identical pions, the intercept $C_{QS}(p,0)$ is still less than 2.

The correlations of non-identical pions also appear to be sensitive to the presence of the quasi–classical source. This sensitivity arises due to properties of the generalized coherent states satisfying the super-selection rules for charged particles, after the averaging over all orientations of the quasi-classical source in isospin space. Due to isospin symmetry of the strong-interaction Hamiltonian, there are unique relations for the intercepts $C^{ij}_{QS}(p,0)$, $i,j=\pi^{\pm},\pi^{0}$, of the pure quantum statistical correlation functions. For example, the coherence suppression of $C^{\pm\pm}$ determines the coherence enhancement of C^{+-}.

In addition to QS, the correlations are also influenced by the two–pion final state interaction (FSI). The latter is well understood and introduces no principle problems. The coherence phenomena can be, however, masked by a number of effects suppressing the measured correlation functions. The most important among them are the decays of long-lived particles and resonances (e.g., Λ, K^0_s, η, η', $\ldots$), the single- and two–track resolution and particle contamination. It is important that the correlations in different two–pion systems are influenced by the QS, FSI and coherence effects in a different way. This offers a possibility to discriminate different effects and so to extract the coherent contribution using correlations functions of like and unlike pions measured at small relative momenta.

In the paper we study the influence of the coherent pion radiation on the behavior of pion inclusive spectra and two–pion correlation functions and, based on it, develop the methods of the extraction of the coherent component above the chaotic background. Despite we associate the coherent radiation with the formation of the quasi-classical field (as the most probable mechanism of the coherence in ultra–relativistic A+A collisions), our results are rather general. Actually, they are based on the general properties of the pion radiation: a common complex source of both the coherent and the chaotic (thermal) radiation, the quasi-classical nature of the coherent pion emitter and the constrains imposed by the charge super–selection rules.

In Sec. II, we set forth the density matrix formalism taking into account the pion emission by partially coherent sources, and the two–pion final state interaction.

In Sec. III, we discuss how to extract the coherent component of particle radiation from the two–pion correlation functions, particularly, in case of large expanding systems produced in ultra-relativistic A+A collisions.

A short summary and conclusion are given in Sec. IV.

II. INCLUSIVE SPECTRA OF PARTIALLY COHERENT INTERACTING PIONS

It is well known that the description of the inclusive pion spectra and two-pion correlations is based on a computation of the following averages [8]:

$$\omega_{\mathbf{p}}\frac{d^3N_i}{d^3\mathbf{p}} \equiv n_i(p) = \sum_{\alpha}|\mathcal{T}(in;p,\alpha)|^2 = \langle a_i^{\dagger}(p)a_i(p)\rangle,$$

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} \equiv n_{ij}(p_1,p_2) = \sum_{\alpha}|\mathcal{T}(in;p_1,p_2,\alpha)|^2 = \langle a_i^{\dagger}(p_1)a_j^{\dagger}(p_2)a_i(p_1)a_j(p_2)\rangle, \tag{1}$$

$$C(p,q) = n(p_1,p_2)/n(p_1)n(p_2),\ \omega_{\mathbf{p}_i} = \sqrt{m^2+\mathbf{p}_i^2},$$

where $\mathcal{T}(in;p,\alpha)$ is the normalized invariant production amplitude. The summation is done over all quantum numbers α of other produced particles, including integration over their momenta; $a_i^{\dagger}(p)$ and $a_i(p)$ are respectively the creation and annihilation operators of asymptotically free pions $i=\pi^{\pm},\pi^0$, the bracket $\langle\ldots\rangle$ formally corresponds to the averaging over some density matrix $|f\rangle\langle f|$.

A special attention requires the production of particles with near-by velocities which can be strongly influenced by the particle final state interaction (FSI). To calculate the FSI effect on particle spectra, we will exploit the density matrix ignoring the long–time scale forces. We will assume sufficiently small phase space density of the produced particles and use the FSI theory in the two–body approximation [8,18,19]. The single–pion spectrum in Eq. (1) then remains unchanged while the two–pion one takes the form

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} \doteq \int d^4k_1 d^4k_2 \langle a_i^{\dagger}(k_2)a_j^{\dagger}(2p-k_2)a_i(k_1)a_j(2p-k_1)\rangle\cdot$$

$$\left[\delta(k_1-p_1)+F^{ij}(k_1,2p-k_1;p_1,p_2)\right]\cdot\left[\delta(k_2-p_1)+F^{ij*}(k_2,2p-k_2;p_1,p_2)\right], \tag{2}$$

where the function F^{ij} is expressed through the propagators of particles i and j and their scattering amplitude $\mathcal{F}^{ij}$ analytically continued to the unphysical region:[3]

$$F^{ij}(k,2p-k;p_1,p_2)=\frac{\sqrt{p^2}}{\pi^3 i}\frac{\mathcal{F}^{ij}(k,2p-k;p_1,p_2)}{(k^2-m^2+i0)[(2p-k)^2-m^2+i0]}. \tag{3}$$

The averaging $\langle\ldots\rangle$ is performed with the help of the statistical operator ρ without FSI, $\langle\ldots\rangle=Sp(...\rho)$. In statistical (hydrodynamic) models of multi-particle production, the density matrix ρ is chosen to be a statistical operator describing the thermal hadronic system in a pre-decaying state on a hyper-surface of thermal freeze-out $\sigma_f: t=t_f(\mathbf{x})$.

The appearance of the quasi-classical pion field $\vec{\pi}_{cl}$ at the thermal stage is usually described in the mean field approximation: $\pi_{i,cl}(x)=\pi_i(x)-\pi_{i,qm}(x)$, where the field $\pi_{i,qm}(x)$ corresponds to the quasi-pion quantum excitations above the temporary vacuum background (the order parameter). Due to the symmetry of the sigma model Lagrangian (see, e.g., [20]), we have $\pi_{i,cl}(x)=e_i\pi_{cl}(x)$, where $\mathbf{e}$ is randomly oriented unit vector , $\mathbf{e}^2=1$, in the three-dimensional isospin space (we suppose that $\mathbf{e}$ does not depend on x). The quasi-classical field created at the chiral phase transition can preserve till the thermal freeze-out stage, particularly, due to the possible effect of *coherence conservation* during the thermal stage of evolution [16]. Then, at the thermal freeze–out, the excitations are distributed according to the Gibbs density matrix with the freeze–out temperature $T\equiv 1/\beta$, e.g. [21]:

$$\rho_{\mathbf{e}}\equiv\rho_{\mathbf{e}}(\sigma_f)\propto\exp[-\int\limits_{\sigma_f}d\sigma_\mu u_\nu\beta T^{\mu\nu}_{\mathbf{e}}(\vec{\pi}_{qm})], \tag{4}$$

where $T^{\mu\nu}_{\mathbf{e}}(\vec{\pi}_{qm})$ is the energy–momentum tensor of free quasi–pions $\vec{\pi}_{qm}$ involved in a collective system expansion with the four–velocity $u(x)$. To calculate the observed inclusive spectra, one has to perform the averaging over the random orientation of the pion quasi-classical field in the isospin space:

$$Sp(...\rho)=(4\pi)^{-1}\int d\Omega(\mathbf{e})\langle\ldots\rangle_{\mathbf{e}}\equiv(4\pi)^{-1}\int d\Omega(\mathbf{e})Sp(\ldots\rho_{\mathbf{e}}). \tag{5}$$

When the decay of such a thermal system happens, the pion quasi-classical field is destroyed and the condensate, or the temporary *disoriented* vacuum, tends to relax back to the normal vacuum by emitting physical pions in (generalized) coherent states. The picture is similar to particle radiation by a classical source. In the absence of an intensive mutual interaction of the pions emitted in such a non-equilibrium transition at the post thermal freeze–out stage, the pions preserve their coherent properties. We will suppose that the quasi-pion masses at thermal freeze-out are near the physical mass, $m_i(t_f)\simeq m_{out}\equiv m$, neglecting a possible mass shift which generates squeeze-state components in particle radiation. We suppose a space–like hypersurface σ_f, and so use the covariant Tomonaga–Schwinger formalism to describe the field operators. An example is the Bjorken hydrodynamic model: $t_f(\mathbf{x})=(\tau^2+x^2_{long})^{1/2}$, where τ is the proper expansion time.

After the thermal freeze-out the system is out of local thermal equilibrium but still can be in a pre-decaying (interacting) state. In fact, the complete decay (neglecting the long-time scale forces) happens at some finite *asymptotic* times $t_{out}<\infty$. We suppose that the system decay at this stage ($t_f<t<t_{out}$) is well approximated by the quantum-field model of the interaction with a classical source [3] (leading to the non-Gaussian density matrix for the quantum states at $t>t_f$), the latter associated with $\pi_{cl}(x)$ [14,16]. Then there is a linear relationship between the annihilation (creation) operators diagonalizing the pion field Hamiltonian at the times t_f and t_{out}:

$$\mathrm{a}_{i,qm}(\mathbf{p},t_{out})=[\mathrm{a}_{i,qm}(\mathbf{p},t_f)+e_i\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out})]e^{-i\omega_{\mathbf{p}}(t_{out}-t_f)}, \tag{6}$$

where c-value quantity $\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out})$ depends on a concrete mechanism and the rate of the classical field decay, and it does not depend on the orientation of the isospin vector $\mathbf{e}$. It follows, from the continuity of the complete field $\pi_i(x)$ and its derivative at the points t_f, that for a fast freeze-out ($t_{out}\to t_f$) the quantity $\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out})$ is

---

[3]It is important that the relation between the production amplitude and the operator product average, as given in Eq. (1), is valid also off mass shell.

directly associated with the strength of the pion condensate. The operators $a_i(p)$ of the asymptotic free pion field (with the origin of the time coordinate shifted to the point t_f) are connected with the operators $\mathrm{a}_{i,qm}(\mathbf{p},t)$ taken at the *asymptotic* times t_{out} by the relation [22]

$$a_i(p) = \sqrt{p_0} e^{ip_0(t_{out}-t_f)} \mathrm{a}_{i,qm}(\mathbf{p}, t_{out}), \quad p_0 = \omega_{\mathbf{p}}. \tag{7}$$

The averages corresponding to the Gaussian type statistical operator $\rho_{\mathbf{e}}$ (guaranteeing $\langle \mathrm{a}_{i,qm}(\mathbf{p}, t_f)\rangle = 0$) with some fixed isospin orientation $\mathbf{e}$ can be evaluated with the help of the usual thermal Wick theorem applied to the operators $\mathrm{a}_{i,qm}(\mathbf{p}, t_f)$ and $\mathrm{a}^{\dagger}_{i,qm}(\mathbf{p}, t_f)$, and using their relation with the asymptotic operators $a_i(p)$ and $a^{\dagger}_i(p)$ according to Eqs. (6), (7). Assuming the pion Compton wave–length much smaller than the typical system lengths of homogeneity (e.g., hydrodynamical lengths) [21] at the thermal freeze-out hypersurface σ_f, and in the absence of the squeeze–state component of pion radiation, we have, for non-identical pions $i \neq j$:

$$\langle a^{\dagger}_i(p_1)a^{\dagger}_j(p_2)a_i(p_1)a_j(p_2)\rangle_{\mathbf{e}} = \langle a^{\dagger}_i(p_1)a_i(p_1)\rangle_{\mathbf{e}}\langle a^{\dagger}_j(p_2)a_j(p_2)\rangle_{\mathbf{e}}, \tag{8}$$

and for identical ones:

$$\begin{aligned}\langle a^{\dagger}_i(p_1)a^{\dagger}_i(p_2)a_i(p_1)a_i(p_2)\rangle_{\mathbf{e}} &= \langle a^{\dagger}_i(p_1)a_i(p_1)\rangle_{\mathbf{e}}\langle a^{\dagger}_i(p_2)a_i(p_2)\rangle_{\mathbf{e}} + \\ \langle a^{\dagger}_i(p_2)a_i(p_1)\rangle_{\mathbf{e}}\langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_{\mathbf{e}} &- \langle a^{\dagger}_i(p_1)\rangle_{\mathbf{e}}\langle a^{\dagger}_i(p_2)\rangle_{\mathbf{e}}\langle a_i(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}.\end{aligned} \tag{9}$$

Here

$$\langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_{\mathbf{e}} = \langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_0 + \langle a^{\dagger}_i(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}, \tag{10}$$

where the irreducible (thermal) part of the two-operator average

$$\langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_0 = \sqrt{p_{10}p_{20}}\langle \mathrm{a}^{\dagger}_{i,qm}(\mathbf{p}_1, t_f)\mathrm{a}_{i,qm}(\mathbf{p}_2, t_f)\rangle_{\mathbf{e}} \tag{11}$$

does not depend on $\mathbf{e}$ in our approximation of the free quasi-pions with $m_i(t_f) \simeq m$, and

$$\langle a_i(p)\rangle_{\mathbf{e}} = e_i d_{coh}(p) \equiv e_i\sqrt{p_0}\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out}) \ , \ e_0 = \cos\theta \ , \ e_{\pm} = \frac{\sin\theta}{\sqrt{2}}e^{\pm i\phi}. \tag{12}$$

Note that the term with minus sign in Eq. (9) is responsible for the suppression of the like pion correlation functions. Physically, this suppression is related to the presence, in the system, of the quasi–classical field $\vec{\pi}_{cl} \neq 0$ in some virtual sense - the *observable* field at the thermal freeze–out stage is related to the ensemble of events only, and it vanishes: $\langle \vec{\pi}_{cl} \ (t_f)\rangle = 0$. Similarly vanish the complete averages of the asymptotically free operators of the pion fields, for example, $\langle a_+(p)\rangle = (4\pi)^{-1}\int d\Omega(\mathbf{e})\langle a_+(p)\rangle_{\mathbf{e}} = 0$. Analogous result one obtains when evaluating the average of the pion field with $\rho = |c\rangle\langle c|$, where $|c\rangle$ are charge constrained coherent pion states (the states with a fixed electric charge and isospin) [7,8] or generalized coherent states. For instance, at zero temperature the density matrix ρ can be expressed via appropriate weighted sum of the corresponding projection operators. As to the matrix density $\rho_{\mathbf{e}}$, at zero temperature it can be expressed through the projections on the usual coherent states for each real component π_1, π_2 or π_3 of the pion field. Since these states are not eigenfunctions of charge operator (only the average charge is equal to zero) they cannot be used as asymptotic states. One could treat them as the states describing the quasi-particles in grand canonical ensemble (an ensemble with variable total charge) at a stage of the chiral symmetry violation. The matrix $\rho_{\mathbf{e}}$ is thus relevant for the ensemble of events with a fixed average condensate $\vec{\pi}_{cl}$. In the paper, we relate the density matrix $\rho_{\mathbf{e}}$ with the true inclusive observables introducing the averaging over the direction of the isovector $\mathbf{e}$ according to Eq. (5).

Let us express the inclusive observables through the chaotic (ch) and coherent (coh) components of the Wigner function

$$f_{\mathbf{e},i}(x,p) = f_{ch}(x,p) + |e_i|^2 f_{coh}(x,p) \tag{13}$$

integrated over the freeze-out hyper-surface σ:

$$\begin{aligned}\langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_{\mathbf{e}} &= \int d\sigma_\mu p^\mu e^{-iq\cdot x} f_{\mathbf{e},i}(x,p),\\ \langle a^{\dagger}_i(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}} &= |e_i|^2 d^*_{coh}(p_1)d_{coh}(p_2) = |e_i|^2\int d\sigma_\mu p^\mu e^{-iq\cdot x} f_{coh}(x,p),\\ \langle a^{\dagger}_i(p_1)a_i(p_2)\rangle_0 &= \int d\sigma_\mu p^\mu e^{-iq\cdot x} f_{ch}(x,p).\end{aligned} \tag{14}$$

Then, we get for the single–pion spectra ($i=\pi^{\pm},\pi^0$):

$$\omega_{\mathbf{p}}\frac{d^3N_i}{d^3\mathbf{p}}=(4\pi)^{-1}\int d\Omega(\mathbf{e})\int d\sigma_\mu p^\mu f_{\mathbf{e},i}(x,p)=\int d\sigma_\mu p^\mu f(x,p),$$
$$f(x,p)=f_{ch}(x,p)+\frac{1}{3}f_{coh}(x,p). \tag{15}$$

Note that the coherent part of the single–pion spectrum

$$\omega_{\mathbf{p}}\frac{d^3N_{coh}}{d^3\mathbf{p}}\equiv\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)=\frac{1}{3}|d_{coh}(p)|^2. \tag{16}$$

To get the two–pion spectra, we have to calculate the 4-operator average in Eq. (2). For identical pions $i=j$, using Eqs. (9), (10) and (13), we get:

$$\begin{gathered}\langle a_i^\dagger(k_2)a_i^\dagger(2p-k_2)a_i(k_1)a_i(2p-k_1)\rangle_{\mathbf{e}}=\int d\sigma_\mu(x_1)d\sigma_\nu(x_2)\times\\ \{\tfrac{k_1^\mu+k_2^\mu}{2}\tfrac{4p^\nu-k_1^\nu-k_2^\nu}{2}e^{i(k_1-k_2)\cdot x_1}e^{-i(k_1-k_2)\cdot x_2}f_{\mathbf{e},i}\left(x_1,\tfrac{k_1+k_2}{2}\right)f_{\mathbf{e},i}\left(x_2,\tfrac{4p-k_1-k_2}{2}\right)+\\ \tfrac{2p^\mu+k_1^\mu-k_2^\mu}{2}\tfrac{2p^\nu-k_1^\nu+k_2^\nu}{2}e^{-i(2p-k_1-k_2)\cdot x_1}e^{i(2p-k_1-k_2)\cdot x_2}[f_{\mathbf{e},i}\left(x_1,\tfrac{2p+k_1-k_2}{2}\right)\times\\ f_{\mathbf{e},i}\left(x_2,\tfrac{2p-k_1+k_2}{2}\right)-|e_i|^4f_{coh}\left(x_1,\tfrac{2p+k_1-k_2}{2}\right)f_{coh}\left(x_2,\tfrac{2p-k_1+k_2}{2}\right)]\}.\end{gathered} \tag{17}$$

For non-identical pions $i\neq j$, out of the three terms in Eq. (17), only the first term contributes.

The main contribution to the amplitude $\mathcal{F}^{ij}$ and therefore to the integral over $d^4k_1d^4k_2$ in Eq. (2) gives a narrow region near $k_1\simeq k_2\simeq p_1$. Supposing sufficiently smooth behavior of the Wigner functions within this region, we can take them out of the integral putting in their arguments $k_1=k_2=p_1$ and get for the spectra of two identical pions $i=j$:

$$\begin{gathered}\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\tfrac{d^6N_{ii}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}\doteq\\ \int d\sigma_\mu(x_1)d\sigma_\nu(x_2)p_1^\mu p_2^\nu\left\{f(x_1,p_1)f(x_2,p_2)+[\langle|e_i|^4\rangle-\tfrac{1}{9}]f_{coh}(x_1,p_1)f_{coh}(x_2,p_2)\right\}\left|\psi_q^{ii}(x)\right|^2+\\ \int d\sigma_\mu(x_1)d\sigma_\nu(x_2)p^\mu p^\nu[f(x_1,p)f(x_2,p)-\tfrac{1}{9}f_{coh}(x_1,p)f_{coh}(x_2,p)]\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)\doteq\\ \omega_{\mathbf{p}_1}\tfrac{d^3N_i}{d^3\mathbf{p}_1}\omega_{\mathbf{p}_2}\tfrac{d^3N_i}{d^3\mathbf{p}_2}\left\{\langle|\psi_q^{ii}(x)|^2\rangle+\langle\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)\rangle'+[9\langle|e_i|^4\rangle-2]G(p_1)G(p_2)\langle|\psi_q^{ii}(x)|^2\rangle_{coh}\right\}\end{gathered} \tag{18}$$

and, for the spectra of two non-identical pions $i\neq j$:

$$\begin{gathered}\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\tfrac{d^6N_{ij}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}\doteq\\ \int d\sigma_\mu(x_1)d\sigma_\nu(x_2)p_1^\mu p_2^\nu\left\{f(x_1,p_1)f(x_2,p_2)+[\langle|e_ie_j|^2\rangle-\tfrac{1}{9}]f_{coh}(x_1,p_1)f_{coh}(x_2,p_2)\right\}\left|\psi_q^{ij}(x)\right|^2=\\ \omega_{\mathbf{p}_1}\tfrac{d^3N_i}{d^3\mathbf{p}_1}\omega_{\mathbf{p}_2}\tfrac{d^3N_j}{d^3\mathbf{p}_2}\left\{\langle|\psi_q^{ij}(x)|^2\rangle+[9\langle|e_ie_j|^2\rangle-1]G(p_1)G(p_2)\langle|\psi_q^{ij}(x)|^2\rangle_{coh}\right\},\end{gathered} \tag{19}$$

where the amplitude $\psi_q(x)\equiv\psi_q(x_1-x_2)$ coincides, up to a phase factor $e^{ip(x_1+x_2)}$, with the two–particle Bethe–Salpeter amplitude [18,19]:

$$\psi_q(x)=e^{\frac{1}{2}qx}+e^{-ipx}\int d^4ke^{ikx}F(k,2p-k;p_1,p_2). \tag{20}$$

This amplitude can be usually approximated by its values at equal emission times in the two–particle c.m.s. ($t^*\equiv t_1^*-t_2^*=0$, $\mathbf{r}^*=\mathbf{x}_1^*-\mathbf{x}_2^*$, coinciding with a stationary solution of the scattering problem $\psi_{-\mathbf{k}^*}(\mathbf{r}^*)$;[4] in the two–particle c.m.s. $\mathbf{p}_1^*=-\mathbf{p}_2^*=\mathbf{k}^*$, $\mathbf{q}^*=-2\mathbf{k}^*$, $q_0^*=0$. The averages of the diagonal terms in Eqs. (18) and (19) are defined as

---

[4]This approximation is possible on condition [19] $|t^*|\ll mr^{*2}$ which is usually satisfied for particle production in heavy ion collisions.

$$\langle|\psi_q(x)|^2\rangle = \frac{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p_1^\mu p_2^\nu f(x_1,p_1)f(x_2,p_2)|\psi_q(x)|^2}{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p_1^\mu p_2^\nu f(x_1,p_1)f(x_2,p_2)}, \tag{21}$$

and - similarly, when the averaging is done over the coherent ($\frac{1}{3}f_{coh}$) or chaotic (f_{ch}) part of the Wigner density f. The averages $\langle\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)\rangle'$ of the non-diagonal terms differ by the substitution $p_{1,2}\to p$ in the nominator:[5]

$$\langle\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)\rangle' = \frac{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p^\mu p^\nu f(x_1,p)f(x_2,p)\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)}{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p_1^\mu p_2^\nu f(x_1,p_1)f(x_2,p_2)}, \tag{22}$$

The function $G(p)$ measures the relative coherent contribution:

$$G(p) = \frac{d^3N_{coh}/d^3\mathbf{p}}{d^3N/d^3\mathbf{p}} \equiv \frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)}{\int d\sigma_\mu p^\mu f(x,p)}. \tag{23}$$

Note that the last equality in Eq. (18) follows from the relation

$$\begin{aligned}
&\int d^4k_1 d^4k_2 \langle a_i^\dagger(k_2)\rangle_{\mathbf{e}}\langle a_i^\dagger(2p-k_2)\rangle_{\mathbf{e}}\langle a_i(k_1)\rangle_{\mathbf{e}}\langle a_i(2p-k_1)\rangle_{\mathbf{e}}\cdot\\
&\left[\delta(k_1-p_1)+F^{ii}(k_1,2p-k_1;p_1,p_2)\right]\cdot\left[\delta(k_2-p_1)+F^{ii*}(k_2,2p-k_2;p_1,p_2)\right]\\
&\doteq |e_i|^4\int d\sigma_\mu(x_1)d\sigma_\nu(x_2)p^\mu p^\nu f_{coh}(x_1,p)f_{coh}(x_2,p)\psi_q^{ii}(x)\psi_{-q}^{ii*}(x)\\
&\doteq |e_i|^4\int d\sigma_\mu(x_1)d\sigma_\nu(x_2)p_1^\mu p_2^\nu f_{coh}(x_1,p_1)f_{coh}(x_2,p_2)\left|\psi_q^{ii}(x)\right|^2.
\end{aligned} \tag{24}$$

The coherence is most directly connected with the intercepts $C_{QS}(p,0)$ of the quantum statistical (without FSI) correlation functions [see Eqs. (1), (15), (18) and (19)]

$$\begin{aligned}
C_{QS}^{ii}(p,q) &= 1+[9\langle|e_i|^4\rangle-2]G(p+q/2)G(p-q/2)+\langle\cos(qx)\rangle',\\
C_{QS}^{ij}(p,q) &= 1+[9\langle|e_ie_j|^2\rangle-1]G(p+q/2)G(p-q/2).
\end{aligned} \tag{25}$$

Calculating the averages

$$\langle|e_0|^4\rangle = \frac{1}{5},\quad \langle|e_\pm|^4\rangle = \langle|e_+e_-|^2\rangle = \frac{2}{15},\quad \langle|e_0e_\pm|^2\rangle = \frac{1}{15}, \tag{26}$$

we get for the intercepts:

$$\begin{aligned}
C_{QS}^{++}(p,0) &= 2-\frac{4}{5}G^2(p),\quad C_{QS}^{00}(p,0) = 2-\frac{1}{5}G^2(p),\\
C_{QS}^{+-}(p,0) &= 1+\frac{1}{5}G^2(p),\quad C_{QS}^{+0}(p,0) = 1-\frac{2}{5}G^2(p).
\end{aligned} \tag{27}$$

Particularly, it follows from Eqs. (27) that the decay of the quasi-classical pion field suppresses the correlation function of identical charged pions and enhances the one of non-identical charged pions, the latter effect being by a factor of 4 smaller. For $G^2(p)=1$ the intercepts in Eqs. (27) coincide with those found in Ref. [23] in the case of a strong pion condensation.

One can check that the intercepts, as well as the QS correlation functions at any q, satisfy the relation [24]

$$C_{QS}^{++}+C_{QS}^{+-} = C_{QS}^{00}+C_{QS}^{+0}. \tag{28}$$

This relation follows from the assumed isotopically unpolarized pion emission. It is valid also for the complete correlation functions (with FSI), except for the region of very small $|q|$ where the correlation functions of charged pions are strongly affected by the isospin non-conserving Coulomb interaction.

---

[5] This difference can be neglected provided sufficiently smooth behavior of the Wigner functions within the width of both the FSI and QS effects. Such a smoothness condition is usually well satisfied for the chaotic part of the Wigner density f_{ch} and, except for the case of a strong and sharply behaved coherent contribution $\frac{1}{3}f_{coh}$, also for the total Wigner density $f=f_{ch}+\frac{1}{3}f_{coh}$.

Note that the correlation functions, as well as their QS parts, satisfy the usual normalization condition $C(p,q) \to 1$ at large $|q|$ provided that the coherent part of the Wigner density vanishes with the increasing $|p|$ faster than the chaotic one, i.e. $G(p) \to 0$ at large $|p|$. To get some insight in a possible behavior of the relative coherent contribution $G(p)$, consider a simple Gaussian parameterization of the partial average of the pion annihilation operator:

$$\langle a_i(p)\rangle_{\mathbf{e}} \sim \exp(-R_{coh}^2\mathbf{p}^2), \tag{29}$$

According to Eq. (14), the corresponding Wigner density

$$f_{coh}(x,p) \sim \exp(-2R_{coh}^2\mathbf{p}^2 - \mathbf{x}^2/2R_{coh}^2), \tag{30}$$

so the parameter R_{coh} determines not only the spectrum, but also the characteristic radius of the region of the instantaneous coherent pion emission in accordance with the minimized uncertainty relation $\Delta x \Delta p = \hbar/2$. Let us assume a similar Gaussian parameterization of the chaotic component of the Wigner density in the non-relativistic momentum region:

$$f_{ch}(x,p) \sim \exp(-2R_T^2\mathbf{p}^2 - \mathbf{x}^2/2R_{ch}^2), \tag{31}$$

where $R_T \equiv (4mT)^{-1/2}$ measures the characteristic size of the single–pion emitter (heat de Broglie length) and $R_{ch} \geq R_T$ is the characteristic radius of the region of the chaotic pion emission. In the considered rare gas limit, we then get the correlator

$$\langle \cos(qx)\rangle'_{ch} = \exp(-R^2\mathbf{q}^2), \tag{32}$$

where $R = (R_{ch}^2 - R_T^2)^{1/2} \approx R_{ch}$ represents (in the absence of the coherent contribution) the usual interferometry radius. The coherent fraction

$$\begin{aligned} G(p) &= D(p)/[1+D(p)], \\ D(p) &= \frac{d^3N_{coh}/d^3\mathbf{p}}{d^3N_{ch}/d^3\mathbf{p}} \equiv \frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)}{\int d\sigma_\mu p^\mu f_{ch}(x,p)} \sim \exp\left[-2\left(R_{coh}^2 - R_T^2\right)\right)\mathbf{p}^2\right]. \end{aligned} \tag{33}$$

We see that $G(p) \to 0$ at large $|p|$ on a reasonable condition $R_{coh} > R_T$. In fact, since the quasi–classical pion field is generated by pion interaction, one may expect the coherence radius R_{coh} close to R_{ch} and therefore - close to the interferometry radius R. Assuming $R_{coh} \approx R_{ch}$, one has

$$\langle|\psi_q|^2\rangle \approx \langle|\psi_q|^2\rangle_{ch} \approx \langle|\psi_q|^2\rangle_{coh}, \quad \langle\psi_q\psi^*_{-q}\rangle' \approx \frac{[1+D(p)]^2}{[1+D(p+q/2)][1+D(p-q/2)]}\langle\psi_q\psi^*_{-q}\rangle'_{ch}. \tag{34}$$

One can see that $\langle\psi_q\psi^*_{-q}\rangle' \approx \langle\psi_q\psi^*_{-q}\rangle'_{ch}$ at small $|q|$ or in the expected case of a small coherent fraction $G(p) = D(p)/[1+D(p)]$. Note that for large coherent contribution, $D(p) \gg 1$, a decrease of the correlation function towards unity with the increasing $\mathbf{q}^2$ is conditioned by the chaotic component $\langle\psi_q\psi^*_{-q}\rangle'_{ch}$ starting at $\mathbf{q}^2 \sim R^{-2}\ln D^2(\mathbf{0}) - 4p^2$. At smaller $\mathbf{q}^2$-values, the behavior of the correlation function is essentially flatter due to the q—-dependence of the denominator in Eq. (34). For the extreme case of pure coherent radiation, $D(p) \to \infty$ ($G\{p\} \to 1$), the function $\langle\cos(qx)\rangle'$ tends to unity (the interferometry radii tend to zero) irrespective of the assumption $R_{coh} \approx R_{ch}$, so the QS part of the correlation function tends to a constant for any two–pion system i,j: $C_{QS}^{ij}(p,q) = 9\langle|e_ie_j|^2\rangle$.

The effect of coherent radiation on pion spectra and $\pi^+\pi^+$ and $\pi^+\pi^-$ correlation functions is demonstrated in Figs. 1-3 for different ratios $D = D(\mathbf{0})(R_T/R_{coh})^3$ of the total numbers of coherent and chaotic pions. The plots correspond to simple Gaussian Wigner functions (30), (31) with $R_T \equiv (4mT)^{-1/2} \approx 0.72$ fm ($T = 0.135$ GeV) and $R_{coh} = R_{ch} = 5$ fm. Under the assumption of a common source of coherent and chaotic pions in ultra–relativistic heavy ion collisions, characterized by a typical radius $R \sim 5 - 10$ fm, the coherent component in the spectra is concentrated in rather small momentum region of a characteristic width $(2R)^{-1} \sim 20 - 10$ MeV/c (see Fig. 1).

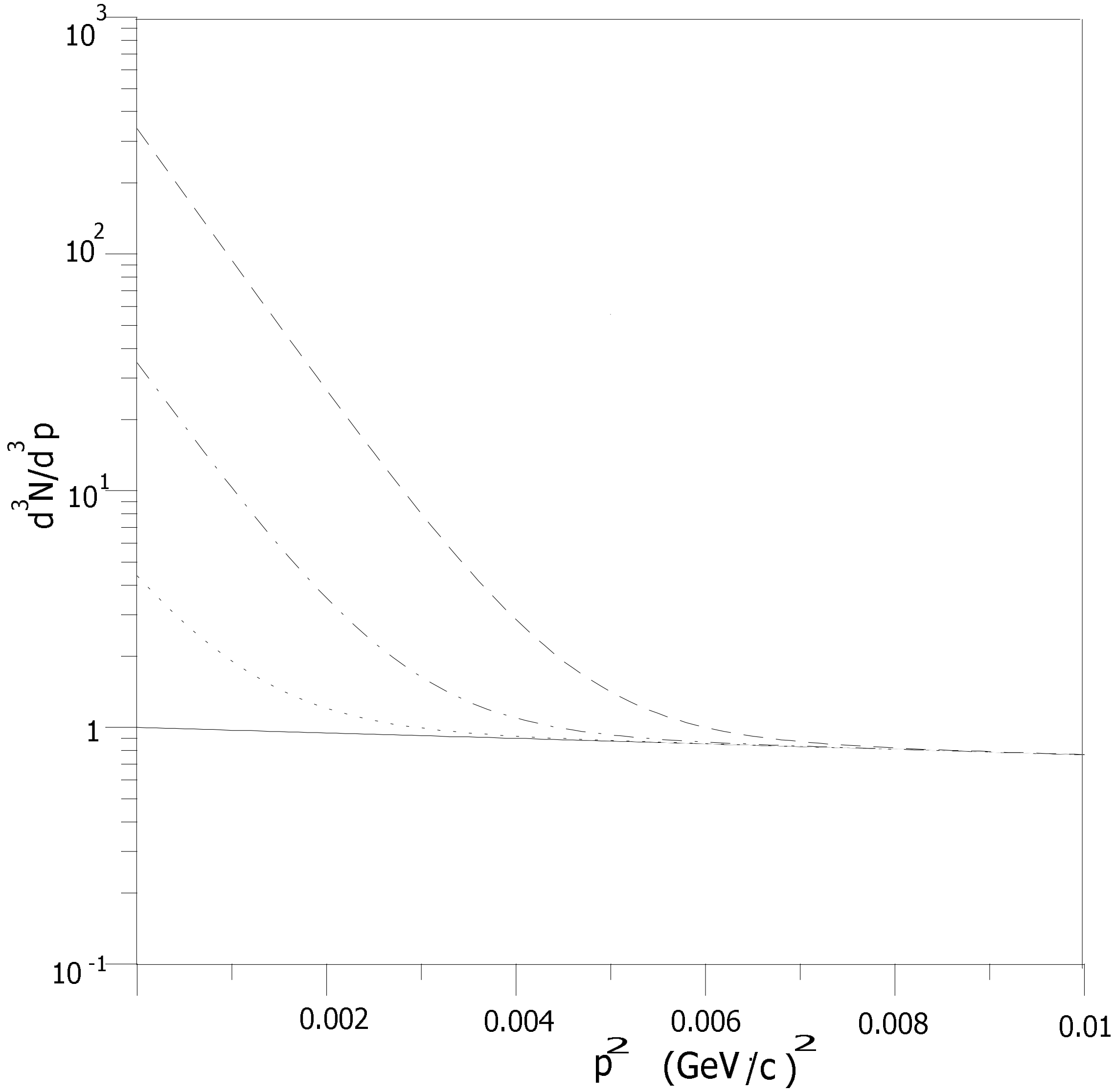

FIG. 1. The single-pion momentum spectra $d^3N/d^3\mathbf{p}$ calculated for different ratios D of the total numbers of coherent and chaotic pions, assuming the Gaussian parameterization of the Wigner densities in Eqs. (30), (31) with $R_T \equiv (4mT)^{-1/2} \approx 0.72$ fm ($T = 0.135$ GeV) and $R_{coh} = R_{ch} = 5$ fm. The solid, dotted, dash-dotted and dashed curves correspond to $D = 0$, 0.01, 0.1, and 1 respectively. The overall normalization is arbitrary.

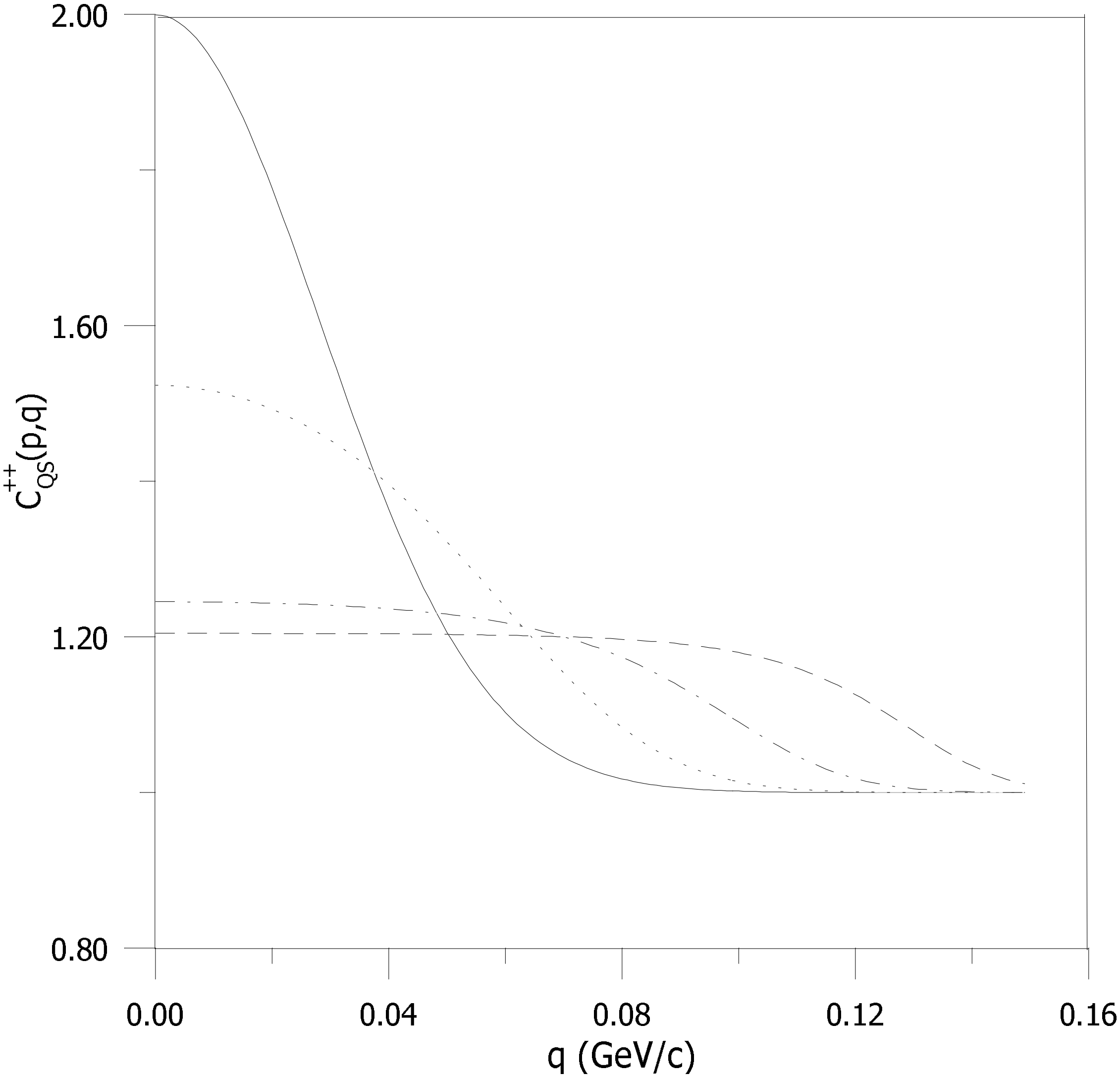

FIG. 2. The pure QS correlation functions $C_{QS}(p, q)$ calculated for $\pi^+\pi^+$ pairs at $\mathbf{p} = \mathbf{0}$ GeV/c on the same conditions as in Fig. 1.

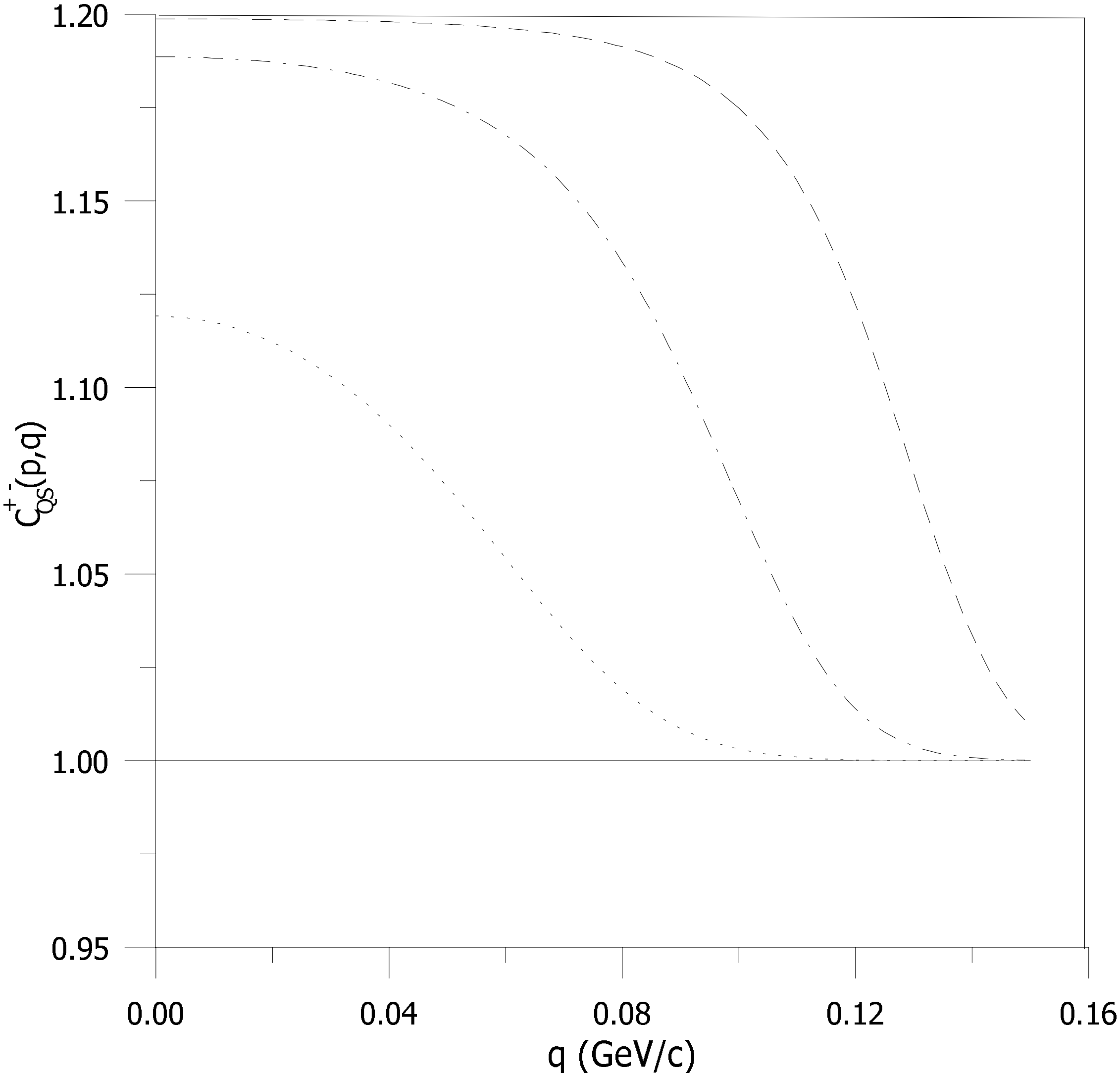

FIG. 3. The pure QS correlation functions $C_{QS}(p,q)$ calculated for $\pi^+\pi^-$ pairs at $\mathbf{p} = \mathbf{0}$ GeV/c on the same conditions as in Fig. 1.

In dynamical models, the interferometry radius varies with the momentum $\mathbf{p}$ and characterizes the size of the homogeneity region - the region of a substantial density of the pions emitted at the freeze–out time with three–momenta $\approx \mathbf{p}$. In the following, we will therefore assume the coherent radius close to the momentum dependent radius $R_{ch}(p)$, allowing to generalize Eq. (34) also for this more realistic case.

III. EXTRACTING COHERENT COMPONENT OF PARTICLE RADIATION

Up to now, we have ignored the contributions $d^3N_i^{(l)}/d^3\mathbf{p}$ arising in the pion spectra from the decays of long–lived (l) sources such as η-, η'–mesons, and also the unregistered K_s^0's and Λ's. The pions from these sources possess no observable FSI (due to very large relative distance of the emission points) as well as no noticeable interference effect (because the corresponding correlation width is much smaller than the relative momentum resolution $q_{\min}$ of a detector). Therefore the measured correlation function defined in Eq. (1) can be expressed through the correlation function $\widetilde{C}^{ij}(p,q)$ of all pion pairs $\pi^i\pi^j$ except for those containing pions from long–lived sources as follows [25]:

$$C^{ij}(p,q) = n_{ij}(p_1,p_2)/n_i(p_1)n_j(p_2) = \Lambda^{ij}(p)\widetilde{C}^{ij}(p,q) + 1 - \Lambda^{ij}(p), \tag{35}$$

where the suppression parameter $\Lambda^{ij}(p)$ measures the fraction of pion pairs containing no pions from long–lived sources:[6]

$$\Lambda^{ij}(p) = \left(1 - \frac{d^3N_i^{(l)}/d^3\mathbf{p}}{d^3N_i/d^3\mathbf{p}}\right)\left(1 - \frac{d^3N_j^{(l)}/d^3\mathbf{p}}{d^3N_j/d^3\mathbf{p}}\right) < 1. \tag{36}$$

In case of absent FSI effect, the correlation function $\widetilde{C}(p,q) = C_{QS}(p,q)$, and the averaging in $\langle\cos(qx)\rangle' \equiv \langle\cos(q(x_1 - x_2))\rangle'$ in the QS correlation functions in Eqs. (25) should be applied only to the pion pairs containing no pions from the long–lived sources. Then, assuming sufficiently good detector resolution, $q_{\min} \ll R^{-1}$, we can determine the intercepts $C(p,0)$ calculating the correlation functions at $|q| \sim q_{\min}$ $(i \neq j)$:

$$\begin{aligned} C^{ii}(p,q_{\min}) &= 1 + \Lambda^{ii}(p)\left\{1 + [9\langle|e_i|^4\rangle - 2]G^2(p)\right\}, \\ C^{ij}(p,q_{\min}) &= 1 + \Lambda^{ij}(p)[9\langle|e_ie_j|^2\rangle - 1]G^2(p). \end{aligned} \tag{37}$$

The intercepts are lower than 2 for any system of identical pions and they are higher (lower) than 1 for $\pi^+\pi^-$ $(\pi^\pm\pi^0)$ systems.

Since the suppression parameters $\Lambda(p)$ are generally different for different pion pairs, e.g., due to different contributions of Λ–decays, it is impossible, using only Eqs. (37), to separate the contributions of the coherent and long-lived sources, unless there is known a ratio of the suppression parameters $\Lambda(p)$ for identical and non-identical pions. Then, for example, from the intercepts of the $\pi^+\pi^+$ and $\pi^+\pi^-$ correlation functions one obtains

$$G^2(p) = \frac{\Lambda^{++}(p)}{\Lambda^{+-}(p)}\left[\frac{4}{5}\frac{\Lambda^{++}(p)}{\Lambda^{+-}(p)} + \frac{1}{5}\frac{C^{++}(p,q_{min}) - 1}{C^{+-}(p,q_{min}) - 1}\right]^{-1}. \tag{38}$$

In fact, the knowledge of the ratio $\Lambda^{ii}(p)/\Lambda^{ij}(p)$ is not of principle importance for the extraction of the fraction $G(p)$. We can exploit the q dependence of $C_{QS}(p,q)$, and perform simultaneous or separate fits of the correlation functions C^{ii} and C^{ij}, suitably parameterizing the correlator $\langle\cos(qx)\rangle'$ and the function $G(p_1)G(p_2)$. For example, we can use the usual Gaussian correlator parameterization

$$\langle\cos(qx)\rangle'_{ch} \simeq \exp(-q_x^2R_x^2 - q_y^2R_y^2 - q_z^2R_z^2) \tag{39}$$

in the longitudinally comoving system (LCMS) in which the pion pair is emitted transverse to the collision axis $(p_L = 0)$. The components of the vector $\mathbf{q}$ are chosen parallel to the collision axis (z=Longitudinal), parallel to the vector $\mathbf{p}_t$ (x=Outward) and perpendicular to the production plane (z,x) of the pair (y=Sideward). Assuming the same radii also for the coherent emission region, and a transverse thermal law $\exp(-m_t/T)$ for the chaotic radiation with the temperature T (m_t is the pion transverse mass), we can parameterize the coherent fraction $G(p)$ similar to Eq. (33) for the non-relativistic case with [16]

$$D(p) \simeq D(0)\exp\left[-2(p_x^2R_x^2 + p_y^2R_y^2 + p_z^2R_z^2) + \frac{m_t}{T})\right], \tag{40}$$

and use Eq. (34) to calculate $\langle\cos(qx)\rangle'$; note that $\psi_q\psi^*_{-q} = \cos(qx)$ in the absence of FSI.

The presence of the FSI effect introduces the additional q dependence of the correlation functions and thus improves, in principle, the accuracy of the extraction of the coherent contribution $G(p)$. Consider, for example, only the effect of the Coulomb FSI and assume that the Wigner functions are localized in the region of a characteristic size much smaller than the two–pion Bohr radius $|a| = 387$ fm. Then the Coulomb effect factorizes in a form of so called Gamow or Coulomb factor $A_c(ak^*) = \left|\psi^{coul}_{-\mathbf{k}^*}(\mathbf{0})\right|^2$ (see, e.g., [8]):

$$\widetilde{C}(p,q) = A_c(ak^*)C_{QS}(p,q), \quad A_c(x) = (2\pi/x)/[\exp(2\pi/x) - 1], \tag{41}$$

where $k^* = |\mathbf{q}^*|/2$ is the momentum of one of the two pions in their c.m.s. For the correlation functions of like $(a = |a|)$ and unlike $(a = -|a|)$ charged pions, we get

---

[6]One can include in $N_i^{(l)}$ and the corresponding suppression parameters Λ^{ij} the contribution of misidentified particles which also introduce practically no correlation.

$$C^{\pm\pm}(p,q) = \Lambda^{\pm\pm}(p)A_c(|a|k^*)\left[1 + \langle\cos(qx)\rangle' - \frac{4}{5}G(p+q/2)G(p-q/2)\right] + [1-\Lambda^{\pm\pm}(p)],$$
$$C^{+-}(p,q) = \Lambda^{+-}(p)A_c(-|a|k^*)\left[1 + \frac{1}{5}G(p+q/2)G(p-q/2)\right] + [1-\Lambda^{+-}(p)]. \quad (42)$$

Similar to the case of absent FSI, we can again use the parameterizations (39) and (40), and fit, simultaneously or separately, the correlation functions of like and unlike charged pions according to Eqs. (42). In addition, the known q dependence of the Gamow factors allows to separate the coherent fraction $G(p)$ from the suppression parameter $\Lambda(p)$ in a model independent way, performing the fits according to Eqs. (42) in an interval of $q_{\min} < |q| \ll R^{-1}$ guaranteeing $\langle\cos(qx)\rangle' \approx 1$ and $G(p_{1,2}) \approx G(p)$. In this case, the correlation functions $C^{\pm\pm}$ are less useful due to their strong Coulomb suppression at very small $|q|$ ($k^* \ll 2\pi/|a|$). On the contrary, the correlation function C^{+-} strongly increases at $|q| \to q_{min}$, and it alone allows to determine the coherent contribution $G(p)$. Of course, such an analysis requires very good detector resolution and its good understanding.

Note that Eqs. (42) are not applicable for very small (~ 1 fm) as well as for large sources. In the former case one has to account for the strong FSI, in the latter - for the finite–size Coulomb effects. For ultra-relativistic heavy ion collisions, the strong FSI is usually negligible, and the Coulomb finite–size effects can be approximately taken into account, substituting the Gamow factor $A_c(ak^*)$ in Eqs. (42) by the finite–size Coulomb factor [26] $\widetilde{A}_c(ak^*, \langle r^*\rangle/a)$. The latter represents a simple function of the arguments ak^* and $\langle r^*\rangle/a$, where $\langle r^*\rangle$ is the mean distance of the pion emission points in the pair c.m.s., corresponding to a given momentum $\mathbf{p}$. Particularly, $\widetilde{A}_c \doteq A_c(ak^*)[1+2\langle r^*\rangle/a]$ at $k^* <\sim \langle r^*\rangle^{-1}$.

The dependence of the Coulomb factor on the unknown parameter $\langle r^*\rangle$ somewhat complicates the model-independent method for the $G(p)$ extraction exploiting only the correlation functions in the region of very small relative momenta. Now, the simultaneous analysis of the correlation functions of like and unlike charged pions is required because their separate analysis yields the coherent contribution $G(p)$ up to a correction $\langle r^*\rangle/a$ only. As for the method based on a fit in a wide $|q|$ interval, the quantity $\langle r^*\rangle$, being a unique function of the parameters characterizing the Wigner density, actually represents no new free parameter. Particularly, for an anisotropic Gaussian $\mathbf{r}^*$–distribution leading to Eq. (39), the quantity $\langle r^*\rangle$ can be expressed analytically through the interferometry radii R_y, R_z and $R_x^* = \frac{M_t}{M}R_x$ (M and M_t are the two–pion effective and transverse masses respectively) in the case of practical interest, when $R_x^* \geq R_y \approx R_z$ [26].

In practice, however, the Gaussian parameterization of the relative distances between the emission points may happen to be insufficient. Particularly, it can lead to inconsistencies in the treatment of QS and FSI effects, the latter being more sensitive to the tail of the distribution of the relative distances. If so, one can no more rely on the equality between $\langle r^*\rangle_{QS}$, determined by the interferometry radii, and the characteristic size $\langle r^*\rangle_C$ determining the Coulomb FSI effect. Generally, one has to introduce also different suppression parameters $\Lambda_{QS} < \Lambda_C$ corresponding to $\langle r^*\rangle_{QS} < \langle r^*\rangle_C$. Eqs. (42) for the correlation functions of like and unlike charged pions, with the substitution of the Gamow factor $A_c(ak^*)$ by the finite–size Coulomb factor $\widetilde{A}_c(ak^*, \langle r^*\rangle/a)$ [26], are then modified to the form:

$$\begin{aligned}
C^{\pm\pm}(p,q) = {} & \Lambda_{QS}^{\pm\pm}(p)\widetilde{A}_c(|a|k^*, \langle r^*\rangle_{QS}^{\pm\pm}/|a|)\left[\langle\cos(qx)\rangle' - \frac{4}{5}G(p+q/2)G(p-q/2)\right] + \\
& \Lambda_C^{\pm\pm}(p)\widetilde{A}_c(|a|k^*, \langle r^*\rangle_C^{\pm\pm}/|a|) + [1-\Lambda_C^{\pm\pm}(p)], \\
C^{+-}(p,q) = {} & \Lambda_{QS}^{+-}(p)\widetilde{A}_c(-|a|k^*, -\langle r^*\rangle_{QS}^{+-}/|a|)\frac{1}{5}G(p+q/2)G(p-q/2) + \\
& \Lambda_C^{+-}(p)\widetilde{A}_c(-|a|k^*, -\langle r^*\rangle_C^{+-}/|a|) + [1-\Lambda_C^{+-}(p)].
\end{aligned} \quad (43)$$

To simplify the analysis, one can neglect a small difference between the suppression parameters Λ_{QS} and Λ_C due to the tail of the r^*–distribution and also neglect a presumably small difference between $\langle r^*\rangle^{\pm\pm}$ and $\langle r^*\rangle^{+-}$.

Finally, after the extraction of the fractions $G(p)$ and $\Lambda^{++}(p)$ or $\Lambda^{--}(p)$, one can obtain the coherent part of the measured single–pion spectra $\omega_{\mathbf{p}} d^3N_\pm/d^3\mathbf{p}$. Using Eq. (36), and substituting $d^3N/d^3\mathbf{p} \to (d^3N_\pm/d^3\mathbf{p} - d^3N_\pm^{(l)}/d^3\mathbf{p})$ in Eq. (23), one gets:

$$\omega_{\mathbf{p}}\frac{d^3N_{coh}}{d^3\mathbf{p}} \equiv \frac{1}{3}|d_{coh}(p)|^2 = \omega_{\mathbf{p}}\frac{d^3N_\pm}{d^3\mathbf{p}}G(p)\sqrt{\Lambda^{\pm\pm}(p)}. \quad (44)$$

The coherent part of the observed spectra is thus directly connected with the intensity $|d_{coh}(p)|^2$ of the quasi-classical source of coherent pions. Would such a source be found by means of the proposed combined studies of $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ correlation functions, the analysis of the pion spectra could give a possibility to discriminate between different mechanisms of coherent production in ultra–relativistic A+A collisions.

IV. CONCLUSIONS

Using the density matrix formalism, satisfying the requirements of the isospin symmetry and the super-selection rule for generalized coherent states, and accounting for the final state interaction in the two–body approximation, we have developed methods allowing one to study the coherent component of pion radiation which, in heavy ion collisions, is likely conditioned by formation of a quasi-classical pion field (DCC ?).

These methods are based on a nontrivial modification of the effects of quantum statistics and final state interaction on two–pion correlation functions (including those of non-identical pions) in the presence of a coherent pion radiation generated by the decay of the quasi-classical pion field. It has been shown that the combined analysis of the correlation functions of like and unlike pions gives the possibility to discriminate the suppressions of the like–pion correlation functions conditioned by the coherent pion component and the decays of long–lived resonances.

The methods allowing to extract the coherent pion component from $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ correlation functions and single–pion spectra have been discussed in detail for large expanding systems produced in ultra–relativistic heavy ion collisions. For such systems, the two–pion final state interaction is dominated by the Coulomb one and plays an important role in this analysis, particularly allowing to determine the coherent fraction in a model independent way provided that sufficiently precise correlation data are available for relative pion momenta much smaller than the inverse size of the system. Alternatively, this fraction can be determined using suitable parameterizations of the coherent and chaotic Wigner densities and fitting the corresponding correlation functions in a wider interval of relative momenta. Such an analysis can be substantially simplified accounting for the finite–size Coulomb effects in an approximate analytic form [26].

Finally, the coherent fractions extracted from the correlation analysis, combined with the single–pion spectra, give us the possibility to determine the spectrum of the coherent pion radiation above the thermal background and therefore to estimate the quasi-classical pion field at the pre-decaying stage of the matter evolution.

V. ACKNOWLEDGMENTS

This work was supported by French-Ukrainian grant No. Project 8917, by Ukrainian - Hungarian Grant No. 2M/125-99, by Ukrainian-German Grant No. 2M/141-2000, and by GA Czech Republic, Grant No. 202/01/0779. We gratefully acknowledge Barbara Erazmus for the interest in this work and fruitful discussions.

---

[1] R.J. Glauber, In: Lectures in theoretical physics. Ed. W.E. Brittin et al., v.1, Interscience Publishers, N.Y., 1959, p. 315; In: High-energy physics and nuclear structure, Proc. of 2nd Int. Conf., Rehovoth, 1967. Ed. G. Alexander, North-Holland, Amsterdam, 1967, p. 311.
[2] Yu.P. Nikitin, I.L. Rosental, High energy nuclear physics, Moscow, Atomizdat, 1980.
[3] E.M. Henley, W. Thirring, Elementary quantum field theory, McGRAW-HILL BOOK COMPANY, 1962; M.I. Shirokov, Yad. Fiz.**7**, 672 (1968).
[4] P. Carruthers, M.M. Nieto, Am. J. Phys. **33**, 537 (1965).
[5] R.J. Glauber, Phys. Rev. **130**, 2529 (1963); Phys. Rev. **131**, 2766 (1963).
[6] D. Horn, R. Silver, Ann. Phys. (N.Y.) **66**, 509 (1971).
[7] J.C. Botke, D.J. Scalapino, and R.L. Sugar, Phys. Rev. D **9**, 813 (1974).
[8] M. Gyulassy, S.K. Kauffmann, L.W. Wilson, Phys. Rev. C **20,** 2267 (1979).
[9] Yu.M. Sinyukov, A.Yu. Tolstykh, Z. Phys. C **61**, 593 (1994).
[10] R. Lednicky, V.L. Lyuboshitz, M.I. Podgoretsky, Sov. J. Nucl Phys. **38**, 147 (1983).
[11] G.Z. Obrant, M.G. Ryskin, Phys. Rev. C **62,** 024903 (2000).
[12] R. Stock, Nucl.Phys. A **661**, 282c (1999).
[13] Proc. of the 15th Int. Conf. on Ultra-Relativistic Nucleus-Nucleus Collisions (Quark Matter 2001), Stony Brook, USA, January 2001 (to be published).
[14] G. Amelino-Camelia, J.D. Bjorken and S.E. Larsson, Phys. Rev. D **56**, 6942 (1997).
[15] A.B. Migdal, Fermions and bosons in intensive fields, Moscow, Nauka, 1978.
[16] S.V. Akkelin, Yu.M. Sinyukov, Nucl. Phys. A **661**, 613c (1999); Heavy Ion Phys. **11**, 47 (2000).
[17] G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. **120**, 325 (1960); G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys. **15**, 219 (1972); G.I. Kopylov, Phys. Lett. B **50**, 472 (1974); M.I. Podgoretsky, Sov. J. Part. Nucl. **20**, 266 (1989).

[18] R. Lednicky, V.L. Lyuboshitz, Heavy Ion Physics, **3,** 1 (1996).
[19] R. Lednicky, V.L. Lyuboshitz, Sov. J. Nucl Phys. **35**, 770 (1982); Proc. Int. Workshop on Particle Correlations and Interferometry in Nuclear Collisions, CORINNE 90, Nantes, France, 1990, p.42, ed. D. Ardouin (World Scientific, 1990).
[20] J. Randrup, Phys. Rev. D **55**, 1188 (1997).
[21] Yu.M. Sinyukov, Nucl. Phys. A **566**, 589c (1994); Heavy Ion Physics, **10,** 113 (1999).
[22] N.N. Bogoliubov, D.V. Shirkov, Introduction to the Theory of Quantized Fields, Wiley, 1980.
[23] C. Greiner, C. Gong, B. Muller, Phys. Lett. B **316**, 226 (1993).
[24] V.L. Lyuboshitz, M.I. Podgoretsky, Yad. Fiz. **21**, 205 (1975).
[25] R. Lednicky, M.I. Podgoretsky, Yad. Fiz. **30**, 837 (1979).
[26] Yu.M. Sinyukov, R. Lednicky, S.V. Akkelin, J. Pluta, B. Erazmus, Phys. Lett. B **432,** 248 (1998).